Number of Distant Cousins

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I have been surprised by the number of contacts with distant cousins through Ancestry. I think part of the reason is that there are many more distant cousins than close cousins. So I thought I would calculate the number of cousins of each degree, based on an average number of descendants per family.

Let $c =$ cousin degree. In this case limited to integers. $c=0$ means siblings, $c=1$ means first cousins etc.

$n =$ number of generations back from me. $n=1$ are my parents, $n=2$ are my grandparents.
The cousin degree is determined by the number of generations back to common ancestors. $c = n-1$

Let $d =$ number of descendants per family who have families and produce more descendants.

Siblings, $c=0$. $n = 1$, Common parents, 1 family.
If there are $d$ descendants per family, then the number of siblings is $d-1$

First cousins, $c = 1$, $n = 2$ Common grandparents. 2 families of grandparents.
My father had $d-1$ siblings. Each of them had $d$ children. Total number of first cousins from my father’s family is $d * (d-1)$
My mother had $d-1$ siblings. Each of them had $d$ children. Total number of first cousins from my mother’s family is $d * (d-1)$
Total number of first cousins is $2d (d-1)$

Second cousins, $c= 2$, $n = 3$ Common great grandparents, 4 families of great grandparents.. Each grandparent had $d-1$ siblings. Each of them had $d$ children of my parents generation, and each of them had $d$ children of my generation.
Total number of second cousins related to any one of my grandparents = $(d-1)^2 * d$ There are 4 of these families, so the number of second cousins is $4 d^2 * (d-1)$

General variable $c$. $n = c+1$ Common ancestors of generation $n$. $2^c$ families with children of the same generation as the common ancestors.
Each of those families produced $(d-1)^c$ children of my generation who are my cousins of degree $c$.
The total number of cousins of degree $c = (2d)^c * (d-1)$

This is intuitively reasonable. If everybody has only 1 child, $d = 1$, and nobody has any cousins of any degree.
If the average is 2 children, then the total number of cousins of degree $c$ is $4^c$. Thus with each increase in degree, the number of cousins of that degree is 4 times as many as the previous degree.
The number of ancestors goes up by a factor of 2 for each generation further back, and the number of descendants from each goes up by a factor of $d$.
So if $d =$ an average of 3 children per generation, the number of cousins of each degree is 6 times as many as the previous degree.
What is a reasonable $d$? Well, it currently is low, but it used to be quite a bit larger. If $d=2$, every 2 people produce 2 people and the population stays the same over time. Since populations actually increase, maybe a long term average might be 3. Some of my ancestors had 4 or more children who produced more children, so the average could be a lot higher.

But even with small $d$, the numbers increase very quickly, as shown in the following table.
If $d = 2$, we have 262 thousand $9^{th}$ cousins.
If $d = 3$, we have 20 million $9^{th}$ cousins….
This is another way of saying we are all related. All of us with British descent, anyway.

This is reminiscent of the problem of counting the grains of rice on a checker board. If you put 1 on the first square, 2 on the second, times 2 on each subsequent square, then ultimately the count is greater than the grain supply.
But in the case of cousins, you put $d-1$ siblings on the first square and multiply by 2$d$ for every subsequent square.
Ultimately the cousin count exceeds the population….. wait a minute… there’s a flaw there somewhere. 😊

Most recently, I heard from a 5$^{th}$ cousin. Our common ancestors had a family of 11. So I must have a huge number of 5$^{th}$ cousins from that family and in the end the probability of hearing from one of them is not as low as I would have previously thought.

For genealogists the message is clear…. The further back you can get, the more likely you will hear from someone…. But everybody knows that anyway…😊
And even if you have few siblings and first cousins that you know about, there are potentially a huge number you have not yet found.

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<th>Number of cousins of degree c</th>
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